

|    |  |  |   |           |  |   |  |  |         |  |  |
|----|--|--|---|-----------|--|---|--|--|---------|--|--|
| 1  | UCL Statistics for Data Sets with Non-Detects  |  |   |           |  |   |  |  |         |  |  |
| 2  |  |  |   |           |  |   |  |  |         |  |  |
| 3  | User Selected Options  |  |   |           |  |   |  |  |         |  |  |
| 4  | Date/Time of Computation   |  | ProUCL 5.110/13/2016 2:30:10 PM           |           |  |   |  |  |         |  |  |
| 5  | From File  |  | ProUCL input 01-006(b) 0-1, 0-5, 0-10.xls |           |  |   |  |  |         |  |  |
| 6  | Full Precision   |  | OFF                                       |           |  |   |  |  |         |  |  |
| 7  | Confidence Coefficient   |  | 95%                                       |           |  |   |  |  |         |  |  |
| 8  | Number of Bootstrap Operations   |  | 2000                                      |           |  |   |  |  |         |  |  |
| 9  |  |  |   |           |  |   |  |  |         |  |  |
| 10 | Plutonium-238  |  |   |           |  |   |  |  |         |  |  |
| 11 |  |  |   |           |  |   |  |  |         |  |  |
| 12 | General Statistics   |  |   |           |  |   |  |  |         |  |  |
| 13 | Total Number of Observations   |  |   | 17        |  | Number of Distinct Observations                                 |  |  | 17      |  |  |
| 14 | Number of Detects  |  |   | 5         |  | Number of Non-Detects   |  |  | 12      |  |  |
| 15 | Number of Distinct Detects   |  |   | 5         |  | Number of Distinct Non-Detects                                  |  |  | 12      |  |  |
| 16 | Minimum Detect   |  |   | 0.0175    |  | Minimum Non-Detect  |  |  | -0.0082 |  |  |
| 17 | Maximum Detect   |  |   | 0.0684    |  | Maximum Non-Detect  |  |  | 0.34    |  |  |
| 18 | Variance Detects   |  |   | 3.6942E-4 |  | Percent Non-Detects   |  |  | 70.59%  |  |  |
| 19 | Mean Detects   |  |   | 0.0375    |  | SD Detects  |  |  | 0.0192  |  |  |
| 20 | Median Detects   |  |   | 0.0337    |  | CV Detects  |  |  | 0.513   |  |  |
| 21 | Skewness Detects   |  |   | 1.201     |  | Kurtosis Detects  |  |  | 1.91    |  |  |
| 22 |  |  |   |           |  |   |  |  |         |  |  |
| 23 | Normal GOF Test on Detects Only  |  |   |           |  |   |  |  |         |  |  |
| 24 | Shapiro Wilk Test Statistic  |  |   | 0.924     |  | Shapiro Wilk GOF Test   |  |  |         |  |  |
| 25 | 5% Shapiro Wilk Critical Value   |  |   | 0.762     |  | Detected Data appear Normal at 5% Significance Level            |  |  |         |  |  |
| 26 | Lilliefors Test Statistic  |  |   | 0.24      |  | Lilliefors GOF Test   |  |  |         |  |  |
| 27 | 5% Lilliefors Critical Value   |  |   | 0.343     |  | Detected Data appear Normal at 5% Significance Level            |  |  |         |  |  |
| 28 | Detected Data appear Normal at 5% Significance Level                                   |  |   |           |  |   |  |  |         |  |  |
| 29 |  |  |   |           |  |   |  |  |         |  |  |
| 30 | Kaplan-Meier (KM) Statistics using Normal Critical Values and other Nonparametric UCLs |  |   |           |  |   |  |  |         |  |  |
| 31 | KM Mean  |  |   | 0.00608   |  | KM Standard Error of Mean                                       |  |  | 0.0065  |  |  |
| 32 | KM SD  |  |   | 0.0233    |  | 95% KM (BCA) UCL  |  |  | 0.0211  |  |  |
| 33 | 95% KM (t) UCL   |  |   | 0.0174    |  | 95% KM (Percentile Bootstrap) UCL                               |  |  | 0.0188  |  |  |
| 34 | 95% KM (z) UCL   |  |   | 0.0168    |  | 95% KM Bootstrap t UCL  |  |  | 0.012   |  |  |
| 35 | 90% KM Chebyshev UCL   |  |   | 0.0256    |  | 95% KM Chebyshev UCL  |  |  | 0.0344  |  |  |
| 36 | 97.5% KM Chebyshev UCL   |  |   | 0.0467    |  | 99% KM Chebyshev UCL  |  |  | 0.0708  |  |  |
| 37 |  |  |   |           |  |   |  |  |         |  |  |
| 38 | Gamma GOF Tests on Detected Observations Only  |  |   |           |  |   |  |  |         |  |  |
| 39 | A-D Test Statistic   |  |   | 0.206     |  | Anderson-Darling GOF Test                                       |  |  |         |  |  |
| 40 | 5% A-D Critical Value  |  |   | 0.681     |  | Detected data appear Gamma Distributed at 5% Significance Level |  |  |         |  |  |
| 41 | K-S Test Statistic   |  |   | 0.175     |  | Kolmogorov-Smirnov GOF  |  |  |         |  |  |
| 42 | 5% K-S Critical Value  |  |   | 0.358     |  | Detected data appear Gamma Distributed at 5% Significance Level |  |  |         |  |  |
| 43 | Detected data appear Gamma Distributed at 5% Significance Level                        |  |   |           |  |   |  |  |         |  |  |
| 44 |  |  |   |           |  |   |  |  |         |  |  |
| 45 | Gamma Statistics on Detected Data Only   |  |   |           |  |   |  |  |         |  |  |
| 46 | k hat (MLE)  |  |   | 5.125     |  | k star (bias corrected MLE)                                     |  |  | 2.183   |  |  |
| 47 | Theta hat (MLE)  |  |   | 0.00732   |  | Theta star (bias corrected MLE)                                 |  |  | 0.0172  |  |  |
| 48 | nu hat (MLE)   |  |   | 51.25     |  | nu star (bias corrected)  |  |  | 21.83   |  |  |
| 49 | Mean (detects)   |  |   | 0.0375    |  |   |  |  |         |  |  |
| 50 |  |  |   |           |  |   |  |  |         |  |  |
| 51 | Estimates of Gamma Parameters using KM Estimates                                       |  |   |           |  |   |  |  |         |  |  |
| 52 | Mean (KM)  |  |   | 0.00608   |  | SD (KM)   |  |  | 0.0233  |  |  |

|     |   |   |   |   |   |           |  |   |   |   |   |        |
|-----|---|---|---|---|---|-----------|--|---|---|---|---|--------|
|     | A   | B | C | D | E | F         | G  | H | I | J | K | L      |
| 53  | Variance (KM)   |   |   |   |   | 5.4105E-4 | SE of Mean (KM)                                |   |   |   |   | 0.0065 |
| 54  | k hat (KM)  |   |   |   |   | 0.0684    | k star (KM)                                    |   |   |   |   | 0.0955 |
| 55  | nu hat (KM)   |   |   |   |   | 2.324     | nu star (KM)                                   |   |   |   |   | 3.247  |
| 56  | theta hat (KM)  |   |   |   |   | 0.089     | theta star (KM)                                |   |   |   |   | 0.0637 |
| 57  | 80% gamma percentile (KM)   |   |   |   |   | 0.00394   | 90% gamma percentile (KM)                      |   |   |   |   | 0.0159 |
| 58  | 95% gamma percentile (KM)   |   |   |   |   | 0.0354    | 99% gamma percentile (KM)                      |   |   |   |   | 0.0988 |
| 59  |   |   |   |   |   |           |  |   |   |   |   |        |
| 60  | Gamma Kaplan-Meier (KM) Statistics  |   |   |   |   |           |  |   |   |   |   |        |
| 61  |   |   |   |   |   |           | Adjusted Level of Significance ( $\beta$ )     |   |   |   |   | 0.0346 |
| 62  | Approximate Chi Square Value (3.25, $\alpha$ )  |   |   |   |   | 0.449     | Adjusted Chi Square Value (3.25, $\beta$ )     |   |   |   |   | 0.36   |
| 63  | 95% Gamma Approximate KM-UCL (use when $n \geq 50$ )  |   |   |   |   | 0.044     | 95% Gamma Adjusted KM-UCL (use when $n < 50$ ) |   |   |   |   | 0.0548 |
| 64  | 95% Gamma Adjusted KM-UCL (use when $k \leq 1$ and $15 < n < 50$ )  |   |   |   |   |           |  |   |   |   |   |        |
| 65  |   |   |   |   |   |           |  |   |   |   |   |        |
| 66  | Statistics using KM estimates on Logged Data and Assuming Lognormal Distribution  |   |   |   |   |           |  |   |   |   |   |        |
| 67  | KM Mean (logged)  |   |   |   |   | N/A       | KM Geo Mean                                    |   |   |   |   | N/A    |
| 68  | KM SD (logged)  |   |   |   |   | N/A       | 95% Critical H Value (KM-Log)                  |   |   |   |   | N/A    |
| 69  | KM Standard Error of Mean (logged)  |   |   |   |   | N/A       | 95% H-UCL (KM -Log)                            |   |   |   |   | N/A    |
| 70  | KM SD (logged)  |   |   |   |   | N/A       | 95% Critical H Value (KM-Log)                  |   |   |   |   | N/A    |
| 71  | KM Standard Error of Mean (logged)  |   |   |   |   | N/A       |  |   |   |   |   |        |
| 72  |   |   |   |   |   |           |  |   |   |   |   |        |
| 73  | DL/2 Statistics   |   |   |   |   |           |  |   |   |   |   |        |
| 74  | Mean in Original Scale  |   |   |   |   | 0.0227    | SD in Original Scale                           |   |   |   |   | 0.0425 |
| 75  | 95% t UCL (Assumes normality)   |   |   |   |   | 0.0407    |  |   |   |   |   |        |
| 76  | DL/2 is not a recommended method, provided for comparisons and historical reasons   |   |   |   |   |           |  |   |   |   |   |        |
| 77  |   |   |   |   |   |           |  |   |   |   |   |        |
| 78  | Nonparametric Distribution Free UCL Statistics  |   |   |   |   |           |  |   |   |   |   |        |
| 79  | Detected Data appear Normal Distributed at 5% Significance Level  |   |   |   |   |           |  |   |   |   |   |        |
| 80  |   |   |   |   |   |           |  |   |   |   |   |        |
| 81  | Suggested UCL to Use  |   |   |   |   |           |  |   |   |   |   |        |
| 82  | 95% KM (t) UCL  |   |   |   |   | 0.0174    |  |   |   |   |   |        |
| 83  |   |   |   |   |   |           |  |   |   |   |   |        |
| 84  | Note: Suggestions regarding the selection of a 95% UCL are provided to help the user to select the most appropriate 95% UCL.              |   |   |   |   |           |  |   |   |   |   |        |
| 85  | Recommendations are based upon data size, data distribution, and skewness.  |   |   |   |   |           |  |   |   |   |   |        |
| 86  | These recommendations are based upon the results of the simulation studies summarized in Singh, Maichle, and Lee (2006).                  |   |   |   |   |           |  |   |   |   |   |        |
| 87  | However, simulations results will not cover all Real World data sets; for additional insight the user may want to consult a statistician. |   |   |   |   |           |  |   |   |   |   |        |
| 88  |   |   |   |   |   |           |  |   |   |   |   |        |
| 89  |   |   |   |   |   |           |  |   |   |   |   |        |
| 90  | Plutonium-239/240   |   |   |   |   |           |  |   |   |   |   |        |
| 91  |   |   |   |   |   |           |  |   |   |   |   |        |
| 92  | General Statistics  |   |   |   |   |           |  |   |   |   |   |        |
| 93  | Total Number of Observations  |   |   |   |   | 17        | Number of Distinct Observations                |   |   |   |   | 17     |
| 94  |   |   |   |   |   |           | Number of Missing Observations                 |   |   |   |   | 0      |
| 95  | Minimum   |   |   |   |   | 0.413     | Mean   |   |   |   |   | 4.933  |
| 96  | Maximum   |   |   |   |   | 24.4      | Median   |   |   |   |   | 3.33   |
| 97  | SD  |   |   |   |   | 5.999     | Std. Error of Mean                             |   |   |   |   | 1.455  |
| 98  | Coefficient of Variation  |   |   |   |   | 1.216     | Skewness                                       |   |   |   |   | 2.349  |
| 99  |   |   |   |   |   |           |  |   |   |   |   |        |
| 100 | Normal GOF Test   |   |   |   |   |           |  |   |   |   |   |        |
| 101 | Shapiro Wilk Test Statistic   |   |   |   |   | 0.731     | Shapiro Wilk GOF Test                          |   |   |   |   |        |
| 102 | 5% Shapiro Wilk Critical Value  |   |   |   |   | 0.892     | Data Not Normal at 5% Significance Level       |   |   |   |   |        |
| 103 | Lilliefors Test Statistic   |   |   |   |   | 0.235     | Lilliefors GOF Test                            |   |   |   |   |        |
| 104 | 5% Lilliefors Critical Value  |   |   |   |   | 0.207     | Data Not Normal at 5% Significance Level       |   |   |   |   |        |

|     |   |   |   |   |        |   |   |   |   |   |   |       |
|-----|---|---|---|---|--------|---|---|---|---|---|---|-------|
|     | A   | B | C | D | E      | F   | G | H | I | J | K | L     |
| 105 | Data Not Normal at 5% Significance Level                                  |   |   |   |        |   |   |   |   |   |   |       |
| 106 |   |   |   |   |        |   |   |   |   |   |   |       |
| 107 | Assuming Normal Distribution  |   |   |   |        |   |   |   |   |   |   |       |
| 108 | 95% Normal UCL  |   |   |   |        | 95% UCLs (Adjusted for Skewness)                                |   |   |   |   |   |       |
| 109 | 95% Student's-t UCL   |   |   |   | 7.473  | 95% Adjusted-CLT UCL (Chen-1995)                                |   |   |   |   |   | 8.211 |
| 110 |   |   |   |   |        | 95% Modified-t UCL (Johnson-1978)                               |   |   |   |   |   | 7.611 |
| 111 |   |   |   |   |        |   |   |   |   |   |   |       |
| 112 | Gamma GOF Test  |   |   |   |        |   |   |   |   |   |   |       |
| 113 | A-D Test Statistic  |   |   |   | 0.387  | Anderson-Darling Gamma GOF Test                                 |   |   |   |   |   |       |
| 114 | 5% A-D Critical Value   |   |   |   | 0.771  | Detected data appear Gamma Distributed at 5% Significance Level |   |   |   |   |   |       |
| 115 | K-S Test Statistic  |   |   |   | 0.155  | Kolmogorov-Smirnov Gamma GOF Test                               |   |   |   |   |   |       |
| 116 | 5% K-S Critical Value   |   |   |   | 0.216  | Detected data appear Gamma Distributed at 5% Significance Level |   |   |   |   |   |       |
| 117 | Detected data appear Gamma Distributed at 5% Significance Level           |   |   |   |        |   |   |   |   |   |   |       |
| 118 |   |   |   |   |        |   |   |   |   |   |   |       |
| 119 | Gamma Statistics  |   |   |   |        |   |   |   |   |   |   |       |
| 120 | k hat (MLE)   |   |   |   | 0.888  | k star (bias corrected MLE)                                     |   |   |   |   |   | 0.77  |
| 121 | Theta hat (MLE)   |   |   |   | 5.556  | Theta star (bias corrected MLE)                                 |   |   |   |   |   | 6.404 |
| 122 | nu hat (MLE)  |   |   |   | 30.18  | nu star (bias corrected)  |   |   |   |   |   | 26.19 |
| 123 | MLE Mean (bias corrected)   |   |   |   | 4.933  | MLE Sd (bias corrected)   |   |   |   |   |   | 5.62  |
| 124 |   |   |   |   |        | Approximate Chi Square Value (0.05)                             |   |   |   |   |   | 15.53 |
| 125 | Adjusted Level of Significance  |   |   |   | 0.0346 | Adjusted Chi Square Value                                       |   |   |   |   |   | 14.67 |
| 126 |   |   |   |   |        |   |   |   |   |   |   |       |
| 127 | Assuming Gamma Distribution   |   |   |   |        |   |   |   |   |   |   |       |
| 128 | 95% Approximate Gamma UCL (use when n>=50)                                |   |   |   | 8.321  | 95% Adjusted Gamma UCL (use when n<50)                          |   |   |   |   |   | 8.807 |
| 129 |   |   |   |   |        |   |   |   |   |   |   |       |
| 130 | Lognormal GOF Test  |   |   |   |        |   |   |   |   |   |   |       |
| 131 | Shapiro Wilk Test Statistic   |   |   |   | 0.943  | Shapiro Wilk Lognormal GOF Test                                 |   |   |   |   |   |       |
| 132 | 5% Shapiro Wilk Critical Value  |   |   |   | 0.892  | Data appear Lognormal at 5% Significance Level                  |   |   |   |   |   |       |
| 133 | Lilliefors Test Statistic   |   |   |   | 0.141  | Lilliefors Lognormal GOF Test                                   |   |   |   |   |   |       |
| 134 | 5% Lilliefors Critical Value  |   |   |   | 0.207  | Data appear Lognormal at 5% Significance Level                  |   |   |   |   |   |       |
| 135 | Data appear Lognormal at 5% Significance Level                            |   |   |   |        |   |   |   |   |   |   |       |
| 136 |   |   |   |   |        |   |   |   |   |   |   |       |
| 137 | Lognormal Statistics  |   |   |   |        |   |   |   |   |   |   |       |
| 138 | Minimum of Logged Data  |   |   |   | -0.884 | Mean of logged Data   |   |   |   |   |   | 0.936 |
| 139 | Maximum of Logged Data  |   |   |   | 3.195  | SD of logged Data   |   |   |   |   |   | 1.255 |
| 140 |   |   |   |   |        |   |   |   |   |   |   |       |
| 141 | Assuming Lognormal Distribution   |   |   |   |        |   |   |   |   |   |   |       |
| 142 | 95% H-UCL   |   |   |   | 14.67  | 90% Chebyshev (MVUE) UCL  |   |   |   |   |   | 10.61 |
| 143 | 95% Chebyshev (MVUE) UCL  |   |   |   | 13.06  | 97.5% Chebyshev (MVUE) UCL                                      |   |   |   |   |   | 16.47 |
| 144 | 99% Chebyshev (MVUE) UCL  |   |   |   | 23.15  |   |   |   |   |   |   |       |
| 145 |   |   |   |   |        |   |   |   |   |   |   |       |
| 146 | Nonparametric Distribution Free UCL Statistics                            |   |   |   |        |   |   |   |   |   |   |       |
| 147 | Data appear to follow a Discernible Distribution at 5% Significance Level |   |   |   |        |   |   |   |   |   |   |       |
| 148 |   |   |   |   |        |   |   |   |   |   |   |       |
| 149 | Nonparametric Distribution Free UCLs                                      |   |   |   |        |   |   |   |   |   |   |       |
| 150 | 95% CLT UCL   |   |   |   | 7.326  | 95% Jackknife UCL   |   |   |   |   |   | 7.473 |
| 151 | 95% Standard Bootstrap UCL  |   |   |   | 7.262  | 95% Bootstrap-t UCL   |   |   |   |   |   | 9.268 |
| 152 | 95% Hall's Bootstrap UCL  |   |   |   | 17.32  | 95% Percentile Bootstrap UCL                                    |   |   |   |   |   | 7.373 |
| 153 | 95% BCA Bootstrap UCL   |   |   |   | 8.327  |   |   |   |   |   |   |       |
| 154 | 90% Chebyshev(Mean, Sd) UCL   |   |   |   | 9.297  | 95% Chebyshev(Mean, Sd) UCL                                     |   |   |   |   |   | 11.27 |
| 155 | 97.5% Chebyshev(Mean, Sd) UCL   |   |   |   | 14.02  | 99% Chebyshev(Mean, Sd) UCL                                     |   |   |   |   |   | 19.41 |
| 156 |   |   |   |   |        |   |   |   |   |   |   |       |

|     | A   | B | C | D | E | F     | G | H | I | J | K | L |
|-----|---|---|---|---|---|-------|---|---|---|---|---|---|
| 157 | <b>Suggested UCL to Use</b>   |   |   |   |   |       |   |   |   |   |   |   |
| 158 | 95% Adjusted Gamma UCL  |   |   |   |   | 8.807 |   |   |   |   |   |   |
| 159 |   |   |   |   |   |       |   |   |   |   |   |   |
| 160 | Note: Suggestions regarding the selection of a 95% UCL are provided to help the user to select the most appropriate 95% UCL.              |   |   |   |   |       |   |   |   |   |   |   |
| 161 | Recommendations are based upon data size, data distribution, and skewness.  |   |   |   |   |       |   |   |   |   |   |   |
| 162 | These recommendations are based upon the results of the simulation studies summarized in Singh, Maichle, and Lee (2006).                  |   |   |   |   |       |   |   |   |   |   |   |
| 163 | However, simulations results will not cover all Real World data sets; for additional insight the user may want to consult a statistician. |   |   |   |   |       |   |   |   |   |   |   |
| 164 |   |   |   |   |   |       |   |   |   |   |   |   |